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For 13 sides, $x^6 - x^5 - 5x^4 + 4x^3 + 6x^2 - 3x - 1 = 0$.

If we form a table of the coefficients of these equations, we discover a curious resemblance to the famous Triangle of Pascal, each column in the triangle gives two columns in our table. The table up to 19 sides is as follows, N being the number of sides of the regular polygon:

N											
3	1	1									
5	1	1	1								
7	1	1	2	1							
9	1	1	3	2	1						
11	1	1	4	3	3	1					
13	1	1	5	4	6	3	1				
15	1	1	6	5	10	6	4	1			
17	1	1	7	6	15	10	10	4	1		
19	1	1	8	7	21	15	20	10	5	1	

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph.D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from the August Number.]

PROPOSITION VI. *The hypothesis of an obtuse angle, if even in a single case it is true, always in every case it alone is true.*

PROOF. Let the join CD (Fig. 5) make obtuse angles with any two equal perpendiculars AC , BD , standing upon any other straight AB .

CD will be (P. III.) less than this AB .

Assume in AC and BD produced any two mutually equal portions CR and DX ; and join RX .

Now I investigate the angles at the join RX , which certainly (P. I.) will be mutually equal.

If they are obtuse we have our assertion.

But they are not right, because thus we would have a case for the hypothesis of right angle, which (P. V.) would have no place for the hypothesis of obtuse angle. But neither are they acute

For thus RX would be (P. III.) greater than this AB ; and still more therefore greater than CD itself. But that this cannot be is thus shown. If the quadrilateral $CDXR$ is taken to be filled up by straight lines cutting off from

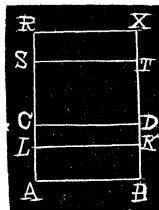


FIG. 5

these CR , DX , portions mutually equal, this implies transition from the sect CD , which is less than AB itself, to RX greater than it, verily transition through a certain ST equal to this AB . But that this is absurd in the present hypothesis follows so; because thus (P. IV.) we have a case for the hypothesis of right angle, which (P. V.) would leave no place for the hypothesis of obtuse angle. Therefore the angles at the join RX must be obtuse.

Then, equal portions AL , BK being assumed in AC , BD ; in a similar manner we show the angles at the join LK cannot be acute toward this AB ; because thus it would be greater than AB , and still more therefore greater than than the sect CD . But here would be found, as above, a certain intermediate between CD less, and LK greater than this AB ; an intermediate, I say, equal to AB itself, which certainly, from what was just now observed, would take away every place for the hypothesis of obtuse angle.

Finally from this very cause the angles at the join LK cannot be right; therefore they will be obtuse.

Therefore with the same base AB , the perpendiculars being increased or diminished at will, the hypothesis of obtuse angle will always persist.

But the same ought to be demonstrated for any assumed base.

Let there be chosen (Fig. 6.) for base any one of the aforesaid perpendiculars, as BX suppose.

Let these AB , RX be bisected in the points M and H ; and MH joined. MH will be (P. II.) perpendicular to these AB , RX . But the angle at the point B is right by hypothesis; and at the point X obtuse, from what has just now been demonstrated.

Make therefore the right angle BXP toward the parts of this MH . XP will meet MH itself in some point P situated between the points M and H ; since on the one hand the angle BXH is obtuse; and, on the other, if XM be joined, the angle BXM (Eu. I. 17.) is acute. Then however, since the quadrilateral $XBMP$ contains three right angles, from what has just now been noted, and one obtuse (Eu. I. 16.) at the point P , because it is external in relation to the internal and opposite right angle at the point H of the triangle PHX ; the side XP will be (Cor. I., P. III.) less than the opposite BM . Wherefore, assuming in BM the portion BP equal to this XP , the angles at the join PP will be (P. I.) mutually equal, certainly obtuse, since the angle BFP (Eu. I. 16.) is obtuse because of the right angle interior and opposite FMP . Therefore the hypothesis of obtuse angle abides for any base BX .

But, as above, this hypothesis abides for this base BX , however much the equal perpendiculars are augmented or diminished at will. Therefore it holds, that the hypothesis of obtuse angle, if even in a single case it is true, always in every case it alone is true.

Quod erat demonstrandum.

Translator's Note. This demonstration assumes that in a quadrilateral like $CDXR$ (Fig. 5.) we can make RX differ from CD as little as we choose

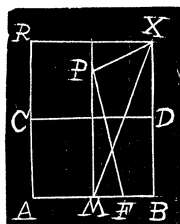


FIG. 6

by making CR and DX sufficiently small; or that the sect ST in passing from coincidence with CD to coincidence with RX is a continuous variable and so passes through all intermediate values.

PROPOSITION VII. *The hypothesis of acute angle, if even in a single case it is true, always in every case it alone is true.*

This is very easily shown. For if the hypothesis of acute angle should permit any case of either other hypothesis, either of right angle, or of obtuse angle, now (from the two preceding propositions) no place would be left for the hypothesis of acute angle; which is absurd.

Therefore the hypothesis of acute angle, if even in a single case it is true, always in every case it alone is true.

Quod erat demonstrandum.

THE FIRST DIFFERENTIAL CO-EFFICIENT OF A CIRCLE.

By Professor JOHN N. LYLE, Ph. D., Professor of Natural Science, Westminster College, Fulton, Missouri.

The equation of the circle AQD , Fig. 1. referred to the Cartesian rectangular co-ordinate axes AY , AX is $y^2 = 2Rx - x^2 \dots (1)$.

Let a point move from A in the path of the curve $AQDK$. Further, let the motion be such that wherever possible dx , the rate of variation of the abscissa x , may be uniform.

y is a function of x , for it takes a new value corresponding to each new value assumed by the variable x .

When the moving point has arrived at F the variable x has reached the individual value x' or Am' ; and the function y , the corresponding individual value y' or $P'm'$.

We then have $y'^2 = 2Rx' - x'^2 \dots (2)$.

Let dx' or $m'B$ stand for the increment to the variable x in the unit of time after reaching the value x' or Am' .

Since x increases uniformly, dx' represents its rate of increase.

Draw BR perpendicular to the axis of abscissas, intersecting the tangent line $TP'R$ in the point R . Draw $P'H$ perpendicular to BR . The line $P'H$ is equal to the constant line $m'B$, and is, consequently, represented by dx .

